A Novel Method for Transmission Line Current Reconstruction in Power Grid

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Current monitoring in power grid is essential for the stability and reliability of the system. We have proposed a novel method for transmission line current reconstruction. Based on regularization process, this method deals with the ill-posed current inverse problem when measurement data is mixed with noise in practical application. The analysis proves the feasibility and robustness of the method compared with two other methods, making it promising in real-time monitoring in power grid.

***Index Terms*—signal reconstruction, power grids, current measurement**

I. Introduction

REAL-TIME monitoring in power grid is essential to keep the stability and reliability of the system. In current measurement, benefiting from advanced sensing techniques such as Hall sensors and GMR/TMR sensors, enormous and abundant data can be achieved for the state monitoring of transmission lines and electrical equipment. However, the reconstruction of current sources from magnetic field measurement results, which can be attributed to the inverse problem between the magnetic field and measured current, still remains an open issue.

Some researchers focus on the reconstruction of currents in transmission lines to realize the state monitoring. The mathematical optimization method has been utilized to calculate the currents [1]. However, since most inverse problems are ill-posed, the traditional optimization method is not suitable when noise is mixed with the magnetic field measurement results. Regularization has been proved to be an effective way to solve an ill-posed inverse problem by introducing additional information. It has been generally applied in the area of medical imaging and geological detection [2].

In this digest, we proposed a novel method to calculate the currents of three-phase transmission lines based on the regularization process. The method and two other methods were analyzed in a practical example to prove the feasibility and stability of regularization.

II. Model Construction

The configuration of a 500 kV transmission line is illustrated in Fig. 1. The reconstruction of current sources in this digest focuses on the calculation in 2D plane, regardless of the sag and galloping of transmission lines. The influence of image current underground is neglected based on the previous research [1].

According to the Ampere's Law, the magnetic field generated by current source I at measurement point P in a 2D plane is indicated in (1).

$$\boldsymbol{h}_0 = \frac{I}{2\pi r^2}\left(\boldsymbol{dl}\times\boldsymbol{r}\right) \tag{1}$$

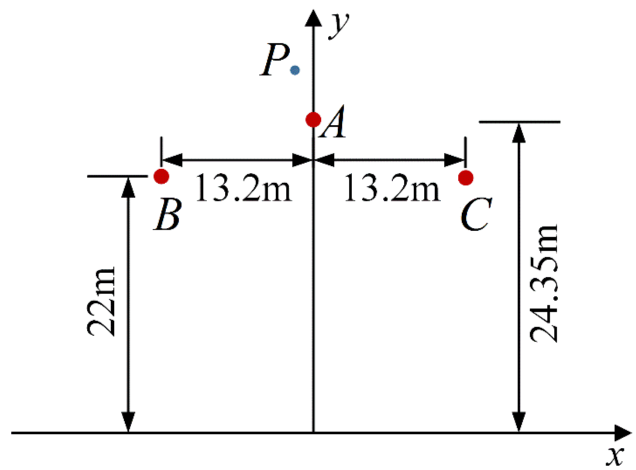

Fig. 1. Configuration of a 500 kV transmission line. *A*, *B* and *C* represent three phases of a 500 kV transmission line. *P* is the measurement point.

where $\boldsymbol{r}$ is the vector from the current source to the measurement point P, $\boldsymbol{dl}$ is the vector of differential current source and is (0,0,1) in this model.

The magnetic field at measurement point P in Fig. 1 is the superimposing of magnetic fields from each phase current, and it is described by (2).

$$\boldsymbol{h} = \frac{1}{2\pi}\sum_{i=1}^{3}\frac{I_i\boldsymbol{dl}_i\times\boldsymbol{r}_i}{r_i^2} \tag{2}$$

The equation can be rewritten as a matrix form in (3).

$$H = Ai \tag{3}$$

where H is the measurement magnetic field vector including two orthogonal components, A is the lead-field matrix determined by the location of current sources and measurement points, i is the current source including the amplitude and phase information.

The reconstruction of current sources is to calculate i according to the lead-field matrix A and measurement results H. In practice, the H vector can be inevitably mixed with noise concerning the limitations of measurement system, and the lead-field matrix A is often ill-conditioned, meaning that even small errors in the measurement results could lead to large calculation errors in current source reconstruction using the traditional solution such as shown in (4) and other optimization method.

$$i = \left(A^T A\right)^{-1} A^T h \tag{4}$$

Tikhonov regularization is an effective and common method to solve the ill-posed problem by introducing additional $\boldsymbol{L_2}$ norm penalty to the optimization process. The minimization problem would be rewritten as in (5).

$$\hat{i} = \arg_I \min\left(\left\|H - Ai\right\|_2^2 + \lambda\left\|Li\right\|_2^2\right) \tag{5}$$

where L is often chosen as a multiple of identity matrix for smoothness, λ is a positive parameter called the regularization parameter.

We choose the proper regularization parameter λ by L-curve method, according to the plot of squared norm of the regularized term $\left\|Li\right\|_2^2$ versus squared norm of the residual vector $\left\|H - Ai\right\|_2^2$ in log-log scale. The above minimization problem in (5) is then equivalent to (6).

$$\hat{i} = \left(A^T A + \lambda L\right)^{-1} A^T H \tag{6}$$

To avoid the fluctuation in one-step Tikhonov regularization, the iteration process is used for a more stable calculation result. Based on the Newton-Raphson method, the numerical iteration process is illustrated in (7).

$$\hat{i}_{k+1} = \hat{i}_k - \left[\left(F'\left(\hat{i}_k\right)\right)^T \left(F'\left(\hat{i}_k\right)\right)\right]^{-1} \left[\left(F'\left(\hat{i}_k\right)\right)^T \left(F'\left(\hat{i}_k\right) - H\right)\right] \tag{7}$$

where $F\left(\hat{i}_k\right)$ is the function of above minimization problem.

The equation can be rewritten as (8) when the function is substituted by the matrix elements.

$$\hat{i}_{k+1} = \hat{i}_k - \left(A^T A + \lambda L\right)^{-1} A^T \left(A\hat{i}_k - H\right) \tag{8}$$

The iteration process stops when the difference between $\hat{i}_{k+1}$ and $\hat{i}_k$ reaches the preset criterion. The current results $\hat{i}_{k+1}$ including the amplitudes and phases of each current phase are the calculation results.

III. Results and Discussion

Based on the above model analysis, a calculation example including two cases was proposed to prove the validity and feasibility of the method. The measurement points were assigned at the height of 28 m, with a span from -3 m to 3 m. The distance between each measurement point was 1 m. The magnetic field vectors were originally calculated according to Ampere's Law at each measurement point. In the reconstruction process, ten groups of calculated magnetic field vectors were mixed with relative random noise between -5% and 5% to simulate the practical measurement situation. The current calculation results using regularization method were compared with the results using (4) and using the simulated annealing (SA) optimization algorithm in MATLAB.

A. Case 1: The balanced loading conditions

In the normal operation state, the currents in transmission lines were respectively set as $1000e^{j0°}$ A, $1000e^{j120°}$ A and $1000e^{j-120°}$A.

The results are illustrated in Table I. The result of direct method has a large relative error of 50% in amplitude due to that the problem is ill-posed, and the result of simulated annealing algorithm is affected by initial value and has a largest amplitude error of 12.4%. The regularization method has a largest amplitude error of 1.3%. It is more accurate and stable concerning the ±5% noise added to the calculated magnetic field vectors, indicating better adaptability under this circumstance.

TABLE I
Current Reconstruction Results of Case 1

Method	Phase	A	B	C
Regularization Method	Current/A	1009.5	999.0	1013.1
	Phase/°	0.62	119	-115
Direct Method Using (4)	Current/A	1111.7	1507.8	1050.6
	Phase/°	-1.82	131	-130
Simulated Annealing Algorithm	Current/A	1053.2	1124.3	1044.1
	Phase/°	0	129	-123

B. Case 2: The unbalanced loading conditions

In the abnormal operation state, the currents in transmission lines were respectively set as $1000e^{j0°}$ A, $800e^{j120°}$ A and $1000e^{j-120°}$A.

The results are illustrated in Table II. The regularization method reconstructs the abnormal current successfully with approximate value. The largest amplitude error is 2.8%. The direct method indicates an obvious difference between the abnormal phase current and the other two phase currents, but the calculated values are inaccurate and has a largest amplitude error of 10.6%. The simulated annealing algorithm has a disadvantage in this case since it is strongly affected by the initial value, the error is even larger and reaches 15%.

TABLE II
Current Reconstruction Results of Case 2

Method	Phase	A	B	C
Regularization Method	Current/A	985	811.2	972.3
	Phase/°	-0.2	120	-119
Direct Method Using (4)	Current/A	1017.0	885.1	1077.7
	Phase/°	0	130	-127
Simulated Annealing Algorithm	Current/A	982.3	921.8	952
	Phase/°	0	110	-115

The above calculation and discussion prove the regularization method as a robust and effective method in practical application for transmission line current source reconstruction in power grid.

IV. Conclusion

A novel method focusing on the current reconstruction of transmission lines has been proposed. The method is based on the Tikhonov regularization process in solving inverse problem. The calculation results indicate the method robust and effective in analyzing measurement data mixed with 5% relative random noise, proving it feasible for practical application in power grid.

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